UNIVERSITY OF CAMBRIDGE
Faculty of Education

ISSN 2043-8338

Journal of Trainee Teacher Education Research

A Critical Analysis of the Value of Investigative Work in the Mathematics Curriculum

Ian Corner

(PGCE Mathematics, 2012-2013)

email: cornerianr@hotmail.com

Abstract

This study looks into the value of investigative work within the secondary mathematics curriculum, specifically why and how it is implemented. This area of mathematics was first highlighted by Cockcroft in the 1980s and has been made further important recently by the National Curriculum and Ofsted. The study is carried out with a high attaining set of year eight boys over a sequence of four lessons. Across these four lessons, two contrasting investigations were used and followed up with tests and focus group interviews. On the whole it was found that: pupils enjoy investigative work, their use can act as a vehicle for greater understanding of mathematical topics, certain syllabus topics can be taught and the teacher develops a less didactic role in investigations and is able to veer towards a social constructivist style of teaching.

A Critical Analysis of the Value of Investigative Work in the Mathematics Curriculum

Ian Corner

Introduction

This study looks at the value of investigative work in the secondary mathematics curriculum, specifically why and how it is implemented. Many hold the belief that the ability to investigate lies at the heart of mathematics (Hunt, 2005) and the now infamous Cockcroft Report (1982) includes in his six aims that mathematics teaching at all levels should include opportunities for investigational work. The current National Curriculum has five attainment targets with the first, Mathematical Processes & Applications (Ma1) dealing directly with process or 'doing mathematics' (Jaworski, 1996). It is further stated that Ma1 should permeate all aspects of the mathematics curriculum (Ball, 1996). Thus, it seems investigations are a vital component of the secondary mathematics curriculum and an area for significant research.

Investigations probably resemble what Skemp (1976) outlines as relational understanding, which concerns what to do and why, compared to instrumental understanding; rules without reasons. These ideas closely resemble constructivist theories of learning. As I begin my teaching career, I feel it is important to establish my own teaching methods and beliefs. I wish to develop relational understanding within my pupils and teach via constructivist methods, helping students construct their own knowledge. Throughout my time in schools so far I have noted some conflict between theory and practice; it can be all too easy for some teachers to slip from a constructivist style of teaching to a more behaviourist style. Constructivism is where learners build on prior knowledge and constantly adapt and develop their structures, the focus being on individual development. By contrast, behaviourism is where the teacher is the dominant person in the classroom, whereby pupils learn by repeated actions and teachers praising correct actions and immediately correcting incorrect ones (Morgan, Watson, & Tikly, 2004). Hence, I wished to look at an investigative style of teaching to assess whether this could help me to achieve my desired goals.

Alongside the aims of the investigations there are other areas to consider, including the role of the teacher which is specifically linked to constructivist and behaviourist theories outlined above.

Furthermore, there is a debate over where investigations sit within the curriculum. Although investigations are included as a national curriculum requirement, there are some issues with aligning investigations with syllabus topics (Jaworski, 1996). Finally, there is the issue of pupils' perceptions of investigation work. All these issues impact on how and why investigation are used in the curriculum and are addressed in this study.

The research consisted of four lessons with a class of top set year eight boys. Two investigations were implemented across these lessons allowing me to assess both my role and the content taught. Before these lessons I spoke with members of the mathematics department to gauge their views and gave an initial questionnaire to pupils to determine their (prior) views. Following the lessons, pupils completed a follow up questionnaire and test to assess their views again and knowledge they had learnt. Certain students were also selected for small focus group interviews to expand on their questionnaire answers.

Within this report, some of the literature surrounding investigational work shall be examined and its influence on the research discussed before an outline on how the research was implemented. Findings will then be presented from the research which will be analysed and discussed before a conclusion is reached on the value of investigations in the mathematics curriculum.

Literature Review

Due to its national curriculum prominence there are many papers published regarding investigations. In this section these shall be reviewed, considering their views on: why investigations are implemented, the role of teachers during them and their place alongside syllabus topics. I will discuss how their findings impact on teaching and learning with relevance to both investigations and more generally. Finally, I shall establish the focus for my research based on my own professional concerns and having been informed by the literature.

Much of the literature was produced following the Cockcroft Report (1982). The report, entitled 'Mathematics Counts' was a committee of enquiry into mathematics teaching and made recommendations for what mathematics teaching should include. Investigational work was identified as one element that should appear at all levels of mathematics teaching.

Ofsted's report (2008) entitled 'Mathematics: Understanding the Score' makes several references to investigations stating they are characteristic of outstanding lessons as they: foster enquiry, develop reasoning, and challenge and extend understanding. Without open-ended tasks and opportunities to engage in 'mathematical talk', Ofsted only grade lessons as satisfactory. If Ofsted rate investigations so highly, it seems likely they are of great value.

Aims & Outcomes of Investigations

Many articles comment on the aims and outcomes of investigations. Firstly, Driver (1988) states that investigations promote more enquiry and retention and less accepting and regurgitation. Jaworski (1996) says investigations promote truly mathematical behaviour and develop processes for use elsewhere. Both articles give reasons why investigations are a national curriculum requirement and seem to agree that investigations can promote greater understanding. Thus, three in-depth research projects shall now be considered to assess whether the intentions lead to the desired results.

Stemn (2008) conducted a sequence of lessons using an investigative approach to teach ratio and proportion. He says giving pupils rules inhibits their chances of using reasoning to solve problems whereas investigations provide these chances. This links with Skemp (1976) who wishes to develop relational understanding. Stemn (2008) states five forms of cognitive activity which support greater understanding: "constructing relationships, applying knowledge, reflecting on experiences, articulating what one knows and making knowledge one's own" (p.384), suggesting that investigations allow this to occur. Through working with the equivalent of year 8 pupils in America, Stemn taught the topic, building on pupils' prior knowledge of fractions. Throughout, pupils were encouraged to: explain their thinking, invent their own strategies, draw on past experiences, and construct solutions before being introduced to standard methods. Stemn believes this helped to deepen their content knowledge and realise the interconnectedness of concepts by considering different strategies. He also states the lessons strengthened skills of: problem solving, communication, representation, proof and reasoning. Stemn's research used a variety of methods giving weight to his claims and was implemented with an equivalent age range to this study and is thus very relevant.

Van Schalkwijk, Bergen and Van Rooij (2000) carried out a project in Dutch education looking at developing students' ideas of proof through investigation. They state that learning results from mental activity by the learner and not through direct transmission. This view is similar to the theory behind Stemn's research and links to constructivism whereby learners build on prior knowledge and constantly adapt and develop their structures (Morgan et al., 2004). The ideas of constructivism revolve around the individual's personal development (Goulding, 2011). It seems that investigations aid in implementing a constructivist teaching manner. Van Schalkwijk et al.'s research is executed with pupils aged 16 and not in the classroom setting. However, their conclusions are valid as the Cockcroft Report (1982) stated that investigations should be carried out at all levels and their theory is similar to Stemn's. The researchers carried out two investigations where everyone actively participated and assisted each other and they conclude that this interplay and critically analysing each other's arguments developed the pupils' competence in proving and thinking mathematically.

Tanner (1989) carried out a project with eight teachers "with the intention of improving teacher performance in the teaching of investigations" (p.264). His main findings include: a) solution strategies are transferable if discovered and not directly taught and b) pupils who experience persistent failure will give up. Thus all pupils must experience 'eureka moments' or they will lose the motivation to persevere. Furthermore, group work helped students to generate and test ideas and communicate results. This communication "seemed to force a clarification of ideas and techniques helping ensure meaningful learning of processes and strategies" (p.266). The idea of transferable skills as highlighted by Stemn (2008) is also discussed by Tanner who suggests that emphasis should be placed on processes and strategies after having solved the problem to allow for generalisation. Tanner also discovered that success depended on whether students accepted mathematical problems as their own challenge to solve, one of Stemn's forms of activity which causes greater understanding. Tanner speaks of the difficulty in getting pupils to make knowledge their own unless there is a real life context to motivate them. A real life context often makes pupils think that a problem is 'worth solving' as the result could be beneficial. They are thus more likely to accept it as their own to solve (Tanner 1989). In general, the greatest effects of Tanner's work were on pupils; they had: a greater sense of achievement, more willingness to explore and argue for validity, an expectation of their mathematics to make sense and a belief they can solve problems.

Overall, the three research projects seem to agree that investigations promote greater understanding and aid a constructivist approach. Tanner (1989) mentions some dangers such as persistent failure and reluctance to accept problems as one's own. Different articles highlight other dangers. Thomas

(1992) says investigations omit vital elements which can cause knowledge gaps and pupils generalise from too few cases showing a lack of proof and a belief in a table of examples. Wells (1995) talks of a scientific view of investigations which emphasise induction only. Pupils do not see analogies, restricting their access to seeing structure and proof. In an earlier article, Wells (1985) speaks of the patterns and rule spotting manner of some investigations whereby pupils experiment, discover and check patterns, and justify or prove results. He again likens this to a scientific approach which is not characteristic and does not model professional mathematicians at work. However, Van Schalkwijk et al. refute Thomas's claim showing the possibility of using investigations to develop proof. Furthermore, Stemn (2008) found that pupils could see structure and interconnectedness countering Well's scientific claims. The 'dangers' spoken about by Thomas and Wells do not appear in the research implemented by Stemn, Van Schalkwijk, et al. or Tanner whose findings are equally backed up by clear research. The 'dangers' outlined seem to arise in the manner an investigation is presented and could potentially be avoided with teacher input. Thus it seems investigations can cause greater understanding, if one is careful as to how they are used.

Teacher Input

Teacher input is another area highlighted by many papers. The Cockcroft Report (1982) states "the teacher needs to help to understand how to apply concepts and skills which are being learned and how to use them to solve problems, both the application to everyday situations within experience and also unfamiliar situations" (paragraph 249). Cockcroft's statement veers towards constructivism whereby the teacher is responsive. Ofsted (2008) found that "teachers encouraged discussion and debate, enabling pupils to learn for themselves and others" (p.43). A detailed observation saw the teacher listening to pupils' explanations, encouraging and nurturing systematic thinking and intervening with additional problems where appropriate. Mini-plenaries were also used to share ideas and stimulate further avenues to explore.

The three research projects discussed previously also comment on the teacher's role. Throughout his work, Stemn (2008) clarified misunderstandings and asked questions to push further, again a reactionary role. Van Schalkwijk et al. (2000) outline the role a teacher possesses, an aid helping to: identify structure, present arguments and distinguish between correct and incorrect arguments. This active intervention allows students to access higher levels. This is a social constructivist approach whereby "knowledge is constructed by means of collaboration but is influenced and regulated by an expert within the so called [Vygotskian] zone of proximal development" (Van Schalkwijk et al., 2000, p.296). A social constructivist approach allows 'meaning making' to develop through language and interaction (Morgan et al., 2004) and interacting with more knowledgeable peers or teachers enables pupils to regulate their own thinking (Goulding, 2011). Despite this theory, the teachers in the project stayed reticent during the first investigation. However, pupils struggled in moving from elementary to advanced mathematics. Hence, in the second round they adjusted this and teachers helped students to understand the need for and validity of proof. Without this, some conjectures would have remained unproven and pupils would not have progressed into advanced mathematics. It seems in investigations that a teacher takes a less didactic role and is more reactionary, responding to pupil need.

Cooper (1990) carried out a project with four Mathematics PGCE students in different schools with different emphases on investigations. Thus, his work is very relevant as these students were in my current position of a PGCE student. All four students experienced a somewhat algorithmic approach during their own schooling, which draws some parallels with mine, as I recall worked examples and textbook practice in the majority of lessons. Thus, it will be interesting to compare these students' experiences to mine and see how they coped with the investigative approach. The first student's school used several approaches. She found, during investigations, "a fair bit of steering" (p.136) was needed and every student needed some help but it was not difficult to be less directive as she was confident in the mastery of the content.

The second student had a less successful experience as he felt he could guide but not help. Investigations carried out at university failed at school due to behaviour issues. Confidence issues drew him towards more algorithmic approaches. Skovmose (2002) states that any investigation challenges the teacher who should not retreat, but find a way to operate in the investigative landscape. Driver (88) is even harsher stating that those who retreat "hide behind their own competence" (p.2) as the teacher's role is to present worthy pieces of mathematics as they still have greater knowledge than their pupils. The third student was in a school where investigations were used constantly and the fourth where they were separated from the rest of the syllabus. Both found discipline issues and that pupils tended to panic if they did not understand and others did. It seems three out of four students lacked confidence in their role and the content.

Tanner (1989) says you need to know “when not to teach” (p.267) which Pritchard (1993) says is a great challenge requiring fine judgement. Tanner (1989) further states teachers should explain less than they think, give pupils think time and not answer their own questions. All articles suggests the teacher’s role is more reactionary using a constructivist or social constructivist approach in response to pupil need to foster greater understanding.

Pupil Perception

A couple of Cooper’s trainees (1990) discovered children prefer more traditional work as they knew what to do. This links to some of Ofsted’s findings (2008); several pupils preferred the routine exercises but some older and more able pupils relished the challenge of investigations. Some pupils indicated they enjoyed group work, disliked too much teacher talk and found textbook work boring and too frequent. Some pupils indicated they wanted more relevant tasks, related to everyday situations. In Van Schalkwijk et al.’s research (2000) they found that “students carried out investigations with satisfaction, experiencing activities as worthwhile” (p.301). This older group of pupils agrees with Ofsted’s findings.

Place in the Curriculum

A further debate is the place of investigations in the curriculum. Cockcroft (1982) suggested that investigations appear at all levels and investigations have their own attainment target. However, Ofsted (2008) found that this attainment target, ‘using and applying mathematics’, was frequently liked to short, real-life problems with rare opportunities to tackle open-ended problems. They found that teachers seldom plan specifically for this strand and that “standards in this crucial aspect remain lower than other areas of the mathematics curriculum” (p.36). This parallels with Jaworski (1996), “many find difficulty with demands of integrating teaching and assessing maths processes and content across the curriculum” (p.8). Ball (1996) instigated a research project at KS3 considering how investigations were used. This study is also at KS3 and so as with Stemn (2008) his work is entirely relevant. His discovered that most schools used investigations less frequently than once a month and less than half the time were they linked to current topics; they were a ‘bolt-on’ activity. He concludes that using investigations in this way is “unlikely to act as a vehicle for process based to permeate” (p.25). Hopkins (1995) states that an on-syllabus/off-syllabus approach is incoherent, concluding that open problems in a closed syllabus is paradoxical. However, Skovmose (2002) suggests some investigations such as sloping squares or the great horse race which fit well into an on-syllabus approach.

Tanner (1989) states that previously some topics were considered so important, they were learnt by rote, highlighting the ongoing debate between teaching basic arithmetic or problem solving. Cooper (1990) found numerous concerns with his PGCE trainees. Trainees using a variety of approaches thought investigations should have purpose; a transmission of definite content which could be reinforced with textbook work. Those using investigations more often were concerned about completing the syllabus and covering important content. This draws parallels with Skovmose (2002) who suggests that "leaving the exercise paradigm in order to explore landscapes of investigation" (p.127) is not the sole answer but we must find a route which allows students to act and reflect. Investigations are a mandatory curriculum requirement but there is an array of opinions of how to best include them to cover syllabus topics.

Research Questions (RQs)

The literature has led the focus for this research. There seems to be three main issues surrounding investigation work: the outcomes of investigation work, the role of the teacher and their place alongside syllabus topics. I am also interested in pupil perception of investigations which is touched upon by Cooper (1990) and Ofsted (2008) and briefly reported above. Thus, out of this literature there are four research questions to consider:

1. What is trying to be achieved by using investigations in lessons?
2. What is the role of the classroom teacher in investigation lessons?
3. What are pupil perceptions about learning mathematics through investigational work?
4. What is the place of investigations within the mathematics curriculum?

These four questions will form the focus of the study and I shall try to answer them with my research and compare it to the literature.

Methodology of my Research

A Case Study Approach

The research uses a case study approach enabling me to study aspects in depth (Bell, 2010) with a detailed narrow focus on individual instances allowing me to combine subjective and objective data (Cohen, Manion & Morrison, 2007) and make analytical rather than statistical conclusions (Robson, 2002). A case study has certain advantages as the events speak for themselves rather than requiring interpretation or evaluation by the researcher (Cohen et al., 2007) and a range of methods can be used (Bell, 2010). Nisbett and Watt (1984) speak of elements of researcher bias and Denscombe (2007) talks of the difficulty in generalising from one instance but advises to identify significant features that could re-occur.

Thus, I deemed this approach appropriate and drew up the following data collection methods (Table 1) which would aid me to answer my research questions.

Data Collection Method	Relevant Research Questions (RQ)
Informal interview members of mathematics department	1, 2 and 4
Initial questionnaire to year 8 class	2 and 3
Sequence of lessons on investigations to year 8 class	1, 2, 3 and 4
Follow up questionnaire and mini test to year 8 class	1, 2, 3 and 4
Group interviews with selected pupils from year 8 class	1, 2, 3 and 4

Table 1 Data collection methods used

Each data collection method shall now be discussed explaining their relevance to the research questions and justifying their method.

Preliminary Staff Interviews

First, preliminary interviews were undertaken with two members of the mathematics department. Staff were selected who had been teaching for a while as I felt they would know more about investigations and how they are best used. By talking to people directly concerned with the topic, I would discover what is of significance to them giving me clues as to what to explore in my own research (Bell, 2010). The intention was to gain initial ideas on the aims of investigations and whether they had strong links to curriculum topics, a direct link to RQs 1 and 4. Experienced teachers would also be able to offer guidance on how to best approach the lessons, linking to RQ3. Cohen et al. (2007) speak of informal conversations as being more relevant as the questions are matched to individuals, but different information is received from multiples sources and analysis varies. Nevertheless, the purpose was to gather initial thoughts to shape my own research and not to make conclusions upon.

The Initial Questionnaire

An initial questionnaire was drawn for the year 8 class, (Appendix 1) intended to ascertain pupils' preliminary perceptions by asking how they felt they best learn and what they enjoy doing in the classroom. Therefore, this questionnaire largely helps to answer RQ3, but some questions could support in answering the other RQs. A questionnaire was used as it would allow me to gather a wide range of opinions quickly and it is more reliable due to anonymity (Cohen et al., 2007). Most of the questionnaire uses closed questions as these prescribe a range of responses so that patterns can be found (Cohen et al., 2007) and comparisons made (Oppenheim, 1992). The questions used a rating scale as from this frequency and correlation can easily be determined (Cohen et al., 2007). Also, the scale has no mid-point so that pupils had to decide either way. However, a questionnaire is not faultless. Rating scales have problems of interpretation as someone's strongly agree could be another's agree (Cohen et al., 2007). Pupils may give falsified replies as they try to complete in a hurry. Furthermore, remarks cannot be added (Oppenheim, 1992) and so responses can only be taken at face value (Bell, 2010) which could be biased. Finally, although I tried my best to avoid this, some questions in my questionnaire could be seen to be loaded with language that could cause pupils to answer in certain ways (Cohen et al., 2007).

After the questionnaire was completed, results were tabulated in a spreadsheet and the mean responses were computed to see whether overall pupils agreed or not with the statements.

The Sequence of Lessons & Investigations

Following the questionnaire, the next four lessons were devoted to actual investigation work. I decided to use two investigations to enable me to make more generalisations from two different instances. The two investigations were both two previous GCSE coursework investigations which I adapted to make more appropriate and relevant for year 8. The first investigation was on the phi function (Appendix 2), lasting for two lessons and homework. The investigation was relatively open allowing pupils to choose their own routes. By contrast, the second investigation on mobile phone tariffs (Appendix 3) was more closed. Cooper (1990) found in his research that different levels of openness have different purposes so I wished to contrast two different types as well. Delivering these investigations would support me in answering all RQs. The lessons' events would enable me to determine what had been achieved by the investigation and what pupils thought of them. I could also carefully consider my role in lessons and whether we had made links to many syllabus topics throughout. During lessons I often jotted down what pupils were doing, conversations I had with pupils or what I was doing and then following the lesson wrote a detailed account of events to aid me with analysis. I also looked over pupil work after each lesson to further study the work they were doing.

The Follow-Up Questionnaire & Test

Following the lessons the pupils completed a follow-up questionnaire and test, (Appendix 4). The questionnaire asked pupils to compare the two different investigations so it could be assessed which one pupils preferred and the different results of both. There was also a short mini-test with questions based on syllabus topics that had been touched on during the two investigations. Thus RQ4 could be answered, where investigations fit in the syllabus. Results of the questionnaire were analysed in the same way as the initial one.

Group Interviews

Finally, group interviews were implemented with a selection of pupils, interviewing nine in groups of two or three for about ten minutes each. I aimed to select a range of pupils who I felt had coped differently with the investigations in order to gain a range of opinions. The main aim was to assess whether pupils' perceptions had changed with a semi-structure of questions posed to pupils considering their: enjoyment, motivation, knowledge gained, confidence in lessons, utilisation of teacher and investigation preference and why. Thus the interviews would help me to answer all RQs. Using similar questions in all interviews would increase comparability (Cohen et al., 2007) and probing questions could be used to investigate motives and feelings (Bell, 2010). Interviews were done in groups as with children it may be less threatening and they can interact (Cohen et al., 2007). Watts and Ebbutt (1987) speak of using group interviews when subjects have been working together on a common purpose as discussions can develop. However, group interviews are not without problems; there is no relevance to individual circumstances (Cohen et al., 2007) as group dynamics may not allow personal matters to emerge (Watts & Ebbutt, 1987). Bassey (1999) says subjects may not fully consider questions, constructing their position on the spot, influenced by the researcher. However, I tried to limit this by implementing interviews straight after the final questionnaire.

Ethical Issues

It was necessary to ensure my research was ethical. Cohen et al. (2007) speak of the necessity of acquiring informed consent from participants, granted from teachers and pupils involved. My research plan was drawn up with and agreed upon by my mentor and the purpose of the lessons was carefully articulated to pupils beforehand with them able to ask questions regarding its nature. Bell (2010) states the researcher should promise anonymity and confidentiality. Thus, questionnaires were all completed anonymously which could also improve reliability of results as pupils may be more willing to write answers they think I do not want. Interviews are not anonymous (Cohen et al., 2007) but are confidential and references to them and pupils' work will be anonymised later in the text. Bell (2010) also says that projects should benefit participants and the school. This research should benefit pupils as the literature points towards investigations having merits and it is concerned with me developing 'best practice' early in my career. The results were shared within the mathematics department and the aims of my research were presented at a staff meeting.

Results of my Research

Within this section, findings will be summaries, first presenting the findings from then initial informal interviews and questionnaires. Then, there will be a summary account of the lessons with some examples of pupils' work before presenting the results of the final questionnaire and interviews.

Staff Discussions

In discussions with staff, I was informed that investigations help pupils with skills of: collaboration, decision making, communication and reasoning skills. However, some investigations can cause pupils to jump to mathematics they do not understand and some may struggle to communicate. Both colleagues gave similar opinions on teaching methods during investigations, saying the teacher is a resource, moving pupils forward, nudging and responding to need. They said investigations work best when learning is student led and everyone chooses their own directions. However, it is a difficult balance deciding whether to consolidate or teach during lessons. Furthermore, you would struggle to teach everything via investigations but there have been attempts to develop syllabi teaching through investigations. Several investigations were deemed safe as they follow a rather formulaic approach, linking to sequences and specific results agreeing with Well's scientific views (1985). Nevertheless, he stated certain investigations bear close resemblance to syllabus topics. All views of teachers agreed with the literature.

Initial Questionnaire Results

The numbers in Table 2 in the boxes below strongly agree, agree etc. show the frequency for that response. The tables' final column is the mean answer for each question.

Table 2 indicates that before the lessons, pupils had quite positive thoughts about investigations. Out of the 26 pupils, 21 indicated that they enjoyed them, felt confident tackling the open style and that by investigating they could increase their understanding. 23 indicated that communicating and discussing would aid their learning and 18 said they enjoyed finding patterns and trying to explain them.

However, all 26 pupils indicated they liked to know the specific topic being studied which may not be clear during investigations and responses varied to discovering new pieces of mathematics; 12 liked it whereas 14 did not. All except one indicated they liked to understand the mathematics they were doing with 22 stating that understanding is more important than getting the right answer. There was a range of responses to teacher use. 24 felt a teacher should help pupils construct knowledge but 20 agreeing it was the teacher's role to give rules and methods, an intriguing paradox. The final question, considering whether investigations should be in the curriculum was agreed upon by 21.

Question	Strongly Agree (1)	Agree (2)	Disagree (3)	Strongly Disagree (4)	Mean Response
1. I learn well by working through textbook exercises on a topic	0	12	13	1	2.58
2. I enjoy doing mathematical investigations	7	14	5	0	1.92
3. I feel confident tackling open problems, not knowing where the work is leading	6	15	5	0	1.96
4. I like to know the specific mathematical topic we are studying	14	12	0	0	1.46
5. Doing investigative work helps to increase my understanding of a topic	8	13	5	0	1.88
6. Discussing ideas and communicating increases my understanding	14	9	3	0	1.58
7. I enjoy discovering new things in mathematics by myself	5	7	12	2	2.42
8. I like spotting patterns and trying to explain them	5	13	8	0	2.12
9. I like to understand the mathematics I am doing	13	12	1	0	1.54
10. Getting the right answer is more important than understanding how to get it	2	2	7	15	3.35
11. My teacher's role is to give me rules and methods to follow	5	15	4	2	2.12
12. My teacher's role is to help me construct my own knowledge	5	19	2	0	1.88
13. Investigations should be part of the mathematics curriculum	7	14	3	2	2.00

Table 2: Summary of responses to initial questionnaire

Question 14 was an open question asking pupils to articulate why they thought investigations were done. I have attempted to group responses together in categories below.

The total frequency exceeds 26, the number of respondents as some pupils gave more than one reason. About a quarter of pupils highlight understanding as a reason. A fifth of pupils think that investigations aid in consolidation, putting mathematics in context and are enjoyable. Four responses indicate that investigations allow for different working styles with two pointing towards group work. It is interesting that pupils came up with these ideas.

Pupil response to 'Why do you think we do investigations?'	Frequency of Response
Broaden Understanding	7
Consolidate Knowledge/Put Into Practice	5
Put Mathematics into Context	5
Enjoyment	5
Keep Occupied	1
Group work	2
Different style of working	4
Work on methods	2
Don't know	2

Table 3: Summary of responses to question 14 on initial questionnaire

Overall it appears that pupils had positive opinions of investigations before the lessons. I will now discuss the lessons, giving a summary account of what occurred.

Account of Investigation Lessons

This first lesson began didactically by introducing the phi function but after that I found myself taking a more reactionary role, listening to what pupils had found and asking them questions to check their work and push them further. I directed some pupils to more systematic ways of working and helped some pupils in understanding and calculating the phi function. Appendix 5 shows an example of systematic working where the pupil has carefully worked through several numbers.

Some pupils adapted really well to the investigation and were keen to explore and discover. Some conjectures (Table 4) show how keen pupils were to explore and the knowledge pupils were touching on, with references to primes, odds and evens. Appendix 6 contains one pupil's work who has made comments in relation to these type of numbers showing the knowledge pupils were considering and evaluating. However, some pupils did not make much progress and left the lesson down hearted due to confusion over the function.

Lesson 1

Timing	Teacher Activity	Pupil Activity
0-10 minutes	The phi function (Φ) is introduced Didactic role in defining the function and key terms: common factor & co-prime Two examples done at board Φ(10) and Φ(16)	Excited and intrigued to no longer be studying algebra and that Φ is a Greek letter Copy down examples from board and try to understand what is happening
10-30 minutes	Told pupils to investigate the phi function with hints to work systematically, try and spot patterns and present findings Clarify with some groups how to calculate and suggest numbers to try – check workings Question pupils as to why prime number result holds, testing/consolidating their knowledge Encouraged pupils to work systematically and not randomly try numbers, discussion of nature of proof	Range of understanding in the class – some pupils needed help in calculating and understanding the phi function Groups working systematically, found prime number result quickly it is one less One group found all results even concluding it must always be even (not true)
30-50 minutes	Whole class intervention. Learning directed to more systematic working so that pupils make discoveries and do not get as frustrated Still some wrong calculations for the phi function which needed correcting Encouraged those who had found patterns to check and explain even if I knew they were wrong	Conjectures begin to form in the classroom: e.g. 'for even numbers, it's half except if divisible by 5' (not true) 'multiples of four, halve the original number' (not true) some genuine excitement 'sir, I've found something'
50-60 minutes	Brought the class together to enable findings and ideas to be shared Chaired the class discussion	Ideas: 'multiples of four it halves' counter example provided by another pupil 'includes up to half the numbers only' (true in some cases) 'one below the number is always in the phi function' (true)

Table 4: Summary account of pupil & teacher activity in lesson one with quotes

Lesson 2

Timing	Teacher Activity	Pupil Activity
0-5 minutes	Phi function reintroduced – ideas shared at end of last lesson displayed on smartboard More structure to investigation given to those who had been struggling. Advised to look at number groups. Those making progress advised to move onto part 2, looking at products.	Listen to recap/explanation
5-25 minutes	Circulate classroom and enter discussion with pupils Help pupils in identifying numbers as powers of two Advised this pair to explore another avenue Still asking questions of pupils and checking calculations	More ideas and conjectures form: 'immediately disregard all the factors' (true) One group spotted powers of two pattern One pair continuing to investigate multiples of four but became frustrated when ideas proved false. One pair looking at multiples of 6. Another pair took this on after hearing conversation with me
25-30 minutes	Whole class intervention - explained what part 2 of the investigation was about in relation to products. Pupils had been having difficulties in understanding what Does $\Phi(n \text{ x } m)= \Phi(n) \text{ x}\Phi(m)$ meant. Couple of examples done to show.	Listen and watch explanation. Try to make sense of what is happening.
30-50 minutes	Re-circulated room and continued to discuss and question pupils Still miscalculations needed correcting. Asked pupils to check their results with more examples and if it worked, try to find an explanation. Guided questioning and hinting got pupils to understand which numbers they needed to check to calculate this and the importance of prime factorisation	Some groups continued looking at groups of numbers and some were looking at products Conjectures begin to form surrounding products 'differ by one it works' (true) 'both even it doesn't work' (true) 'even & odd, it works' (true in some cases) One group trying to calculate $\Phi(44)$
50-60 minutes	Chaired class discussion once more	Ideas again shared as a class

Table 5: Summary account of pupil & teacher activity in lesson two with quotes

The powerpoint slide projected at the lesson's start (Appendix 7), encouraging pupils to take certain avenues for investigation, was aimed at those who had not coped that well in the previous lesson.

Conjectures still developed in the classroom (Table 5). My role depended on pupil need: helping them to decide what to do, questioning or explaining. The intervention staged at 25 minutes helped pupils to progress with the second part, products. Pupils continued utilising knowledge of primes, odds, evens, factors and multiples and Appendix 8 presents one pupil's work who has found specific cases of the general result by finding patterns for products of varying combinations of odd and even numbers.

Via questioning and hinting some pupils accessed much higher levels such as the importance of prime factorisation. Appendix 9 includes one pupil's work showing clearly his systematic way of working on powers and multiples leading to some rules being discovered. However, in all cases pupils have merely found rules but have not been able to explain them without my assistance. There was still genuine excitement in the class with pupils keen to explore and discover. However, some pairs did not progress much beyond initial calculations. Perhaps I should have used a more didactic role and done more modelling to help these pupils as they were frustrated.

The findings from the next two lessons shall now be presented which moved onto the second investigation, the mobile phones task.

The context of the problem had advantages and disadvantages: it motivated and engaged but caused problems as mathematics clashed with real life. My role saw me correcting this. Appendices 10 and 11 show two different approaches to initially solving the problem: the first pupil calculating for a range of months and the second looking only at real life contracts respectively.

The second half of the lesson saw a more didactic role to my teaching with a lot of board work and me directing the pupils' learning as shown in Table 6. Many pupils did not see the point in continuing as the original problem had already been solved. It seemed pupils merely wanted the answers and not methods.

Lesson 3

Timing	Teacher Activity	Pupil Activity
0-5 minutes	Introduced the mobile phone task to pupils. Gave them the hook. Find the cheapest contract.	Excited by the context of the problem and the iPhone
5-15 minutes	Circulated room and helped pupils Explained this was the crux of the problem, time frame dependent Explain we need to assume any length contract can be had	 Some did not understand as they thought needed to know how long I wanted the phone for Some caught up in real life context, calculating only 6 months, 12 months etc.
15-25 minutes	Class brought back together to discuss results and how answers were being achieved Guided questioning to express contracts for *x* number of months	Most replied T-Mobile better in the long run and adding on 10.5 or 20 each time Considering questioning to solve problem for *x* number of months. Some pupils coping well with algebra x times 2 is 2x Some pupils did not see the point in continuing as original problem had been solved
25-45 minutes	Asked pupils how we could use the equations to solve our original problem asking how we could compare two schemes to find where one becomes more economical Directed pupils towards setting equations equal to each other in order to solve Asked pupils the significance of 7.4	Few minutes of ideas from pupils which led to frustration from pupils as not answers I wanted/anticipated Considering how we use the equations to solve the original problem. One pupil came up to board to solve for x, returning 7.4 'one is more expensive after 7.4 months'
45-60 minutes	asked pupils to compare the other schemes to each other Circulated room	Some working well. Some not. One pupils solved for 4.91 and immediately exclaimed 'so it's 5 months'

Table 6: Summary account of pupil & teacher activity in lesson three with quotes

Lesson 4

Timing	Teacher Activity	Pupil Activity
0-10 minutes	Asked pupil to recall equations developed last lesson Displayed a set of axes on the board and asked pupils how we could use this Demo of plotting points	Recall equations. Only a couple did. But others participating once initial ones done Still some grumbling about continuing with this problem 'plot month against tariff' immediate reply from one boy 'they'll form a straight line'
10 -25 minutes	Circulate and assist pupils	Pupils plot the points for the three schemes
20- 30 minutes	Class discussion as to what is noticed	'Some are steeper than others' 'this one is cheaper for the first bit and then this one for the second bit' 'they form a straight line as they go up the same each month'
30-40 minutes	Displayed my graph on board. Extended beyond data range across y-axis. Displayed questions on board for pupils to consider [see appendix 12] Circulated room	Consider questions on the board in relation to the graphs 'they should all pass through origin as after 0 months, you pay 0 pounds; 'they cross the axis at the original price of the phone'
40-50 minutes	Question posed: 'what can we use the graphs for?' Introduced concept of intersection points and asked for relevance to equations	'read off the cost for any time frame' 'where graphs meet, schemes are equal' 'the value you get when solving the equation is the same as the intersection point as it's when they are equal'
50-60 minutes	Returned to the point about graphs being steeper and why? 'Which number in the equation represented the tariff?'	'they have higher tariffs' 'the number multiplying the x' 'graphs are steeper if the number multiplying x is bigger and shallower if smaller' 'they have a higher gradient'.

Table 7: Summary account of pupil & teacher activity in lesson four with quotes

After solving the required equations I am unsure whether pupils could relate this back to the original problem and saw the 'break-even' point as between 7 and 8 months or saw the point of doing this. Those who did could not obviously see which one was cheaper. A lack of connection between the tabular method and equations was evident. Some pupils were lost in the latter half, not appreciating the relevance of discussions. We covered a syllabus topic but with steering, just like one of Cooper's PGCE students discovered (1990). However, some pupils were eager and afterwards discussed with me how we used equations further.

At the beginning several pupils were reluctant to continue but all pupils seemed to enjoy plotting the graphs. Again learning was rather teacher led but pupils had some great insight and conjectures during class discussions (Table 7) on my questions on the PowerPoint slide (Appendix 12).

The issue of context in mathematics problems was prevalent again as shown by one pupil who felt the graphs should all pass through the origin. An example of the graphs drawn is included in Appendix 13. Nearly all graphs resembled this. Thus, I was directing learning and not letting pupils be free and make their own decisions.

Some pupils saw the links between the graphical representation and equations covered in the previous lesson showing they understood the interconnectedness of ideas. A quick straw-poll revealed most preferred the graphical method.

This second lesson on Mobile Phones was more successful than the first as pupils coped better with the graphs than the equations. We covered a clear syllabus topic, albeit with guidance from me. However, pupils were still discovering for themselves. Despite this, in class wide discussions I cannot be sure all pupils were actively involved as it was often the same pupils contributing.

Test and Questionnaire Results

The results of the concluding test and questionnaire shall now be discussed which asked pupils to compare the two investigations and tested them on knowledge gained throughout.

Which Investigation was Preferred and why?			
Number Preferring Phi Function		Number preferring Mobile Phones	
7		19	
Reason for preferring phi	Number of respondents	Reason for preferring Mobile Phones	Number of respondents
Learnt more	1	More fun	3
More to explore	2	More interesting	1
Interesting	2	Easier to understand	5
More challenging	1	Useful in real life	4
More fun	1	More to learn	1
		Phi didn't make sense	2

Table 8: Summary of responses to questionnaire asking pupils to state investigation preferred and why

Table 8 shows 19 pupils preferred the mobile phone investigation, mainly because it was easier and more fun and useful than the phi investigation. The 7 who preferred phi indicated they enjoyed the openness and challenge of it.

Questions expressing opinions on the Phi Function Investigation	Strongly Agree (1)	Agree (2)	Disagree (3)	Strongly Disagree (4)	Mean Response
1. I coped well with this investigation	4	10	5	5	2.46
2. The openness of the problem appealed to me	6	9	5	4	2.29
3. It was easy to make progress with this investigation	3	10	4	7	2.63

Table 9: Summary of responses to questionnaire asking pupils to state their opinions on the phi investigation

Table 9 above suggests a wide array of opinions on the phi function investigation throughout the class. 14 felt they coped well and enjoyed it whereas 10 strongly felt they had not. 15 liked the openness compared to 9 who did not and 13 felt they made progress compared to 11 who did not. It seems the class is divided in their opinions on this investigation.

Question	Answer	Number of Respondents
What does co-prime mean?	HCF is 1 *	12
	HCM is 1	1
	One prime to the other	1
	Factors are 1 and self	1
	Neither go into a number	1
	No answer	9
What is Φ(n) if n is prime?	n-1 *	13
	all numbers below *	1
	no answer	11
Does Φ(4x3) = Φ(4) x Φ(3) ? Why?	Yes, no reason	2
	Yes, odd and even (true in some cases)	4
	Yes, one difference *	1
	Yes, 3 isn't a factor of 4 (true in some cases)	1
	Yes, they're co-prime *	2
	No	4
	No answer	11
Does Φ(4x2) = Φ(4) x Φ(2) ? Why?	No, no reason	2
	No, both even *	3
	No, calculation done *	3
	No, it will be half *	1
	No, 2 is a factor of 4 *	1
	No, not co-prime *	1
	Yes, both even	1
	Yes, no reason	2
	No answer	11
When does Φ(mxn) = Φ(m) x Φ(n) ?	When one difference in numbers *	2
	When one is prime (true in some cases)	2
	When one odd and one even (true in some cases)	1
	When co-prime to each other *	3
	No answer	17
How is prime factorisation important?	To see if there are co-primes	2
	No answer	23

Table 10: Summary of responses to questions testing pupils on knowledge gained during phi function investigation with acceptable answers indicated by *

It was difficult to create a test for this investigation as most questions dwell on the phi function itself and discoveries made, not more general mathematics. However, from numerous pupils' responses (Table 10) it is evident they have used their mathematical knowledge throughout. Several comments refer to primes, factors, odds, evens and co-primes. It is likely that this investigation consolidated knowledge of these. However, several questions are unanswered suggesting some did not cope well or learn much from the investigation.

Questions expressing opinions on the Mobile Phones Investigation	Strongly Agree (1)	Agree (2)	Disagree (3)	Strongly Disagree (4)	Mean Response
1. I coped well with this investigation	11	12	2	0	1.64
2. It was easy to make progress with this investigation	11	12	1	1	1.68
3. I liked the context of the problem	12	10	2	1	1.68
4. I was satisfied that the investigation had a solution	8	9	5	2	2.04
5. I saw little point in continuing when we had already solved the initial problem	5	11	8	1	2.20

Table 11: Summary of responses to questionnaire asking pupils to state their opinions on the mobile phones investigation

Table 11 indicates overall, opinions of this investigation were better. More pupils thought they coped well with this investigation, 23 compared to 14 for phi. More also indicated they made better progress, 23 compared to 12 for the phi function. 22 pupils enjoyed the context as well. Questions four and five were included based on personal observations to which responses vary; 17 liked that there was a solution, 7 did not, 16 wanted to continue, 9 did not. There is no clear-cut response to this question; more work could perhaps be undertaken in this area.

Question	Answer		Number of Respondents
Which of these equations will produce the steepest line graph?	3x+4		0
	2x+15		11
	9x+1 *		13
Why is this graph steepest?	Those who answered 2x+15	Biggest Tariff	1
		Goes up Quickest	1
		No Answer	11
	Those who Answered 9x+1	Biggest	1
		Times is highest *	2
		Start on graph	1
		Goes up 9 and will catch up*	2
		Added to total	1
		No Answer	6
What is the relevance of the numbers 4, 15 and 1 in the above?	Integers		1
	Tariff		5
	Added at beginning *		2
	Which one will rise quicker		1
	Original Price of phone *		5
	No Answer		11
What is significant about points where lines on a graph intersect?	Point of Intersection		2
	One becomes cheaper *		3
	Find out what x is		1
	Where they're equal *		6
	No answer		10
Without using a graph how else could you find out where they would intersect?	Looking		1
	Using maths		1
	Use a Table *		2
	Equations *		4
	Internet		1
	You can't		1
	No answer		14

Table 12: Summary of responses to questions testing pupils on knowledge gained during mobile phones investigation with acceptable answers again indicated by a *

This test was easier to design as more 'general' mathematical topics had been covered in lessons, solving linear equations and straight line graphs. It is interesting that questions (Table 12) do not refer to mobile phones, yet numerous responses refer to tariffs and phone prices. The knowledge may be tied to the problem's context. Only about half the pupils correctly answered questions concerning gradients and y-intercepts. Several pupils have them confused or have not answered. Perhaps students were not actively involved in class discussions or not enough consolidation was done. The last question to which 6 replied 'equations' or 'a table' shows some pupils see links between the different forms of representation. Thus, it seems some pupils have learnt some syllabus mathematics through this investigation.

Interview Results

Nine pupils were interviewed in four groups, believed to have coped differently with the investigation. Table 13 includes responses from those perceived to adapt well to the investigations and Table 14 those perceived to not adapt as well or struggled in certain areas. The topics listed in column one were the question areas pupils were asked them to expand upon.

	Responses from Different Groups	
Question Area	Group 1 – Coped Well – Good insight – Less contribution to class discussion	Group 2 – Coped Well and a lot of contribution to class discussions
Enjoyment	Fun, good. Were able to discuss, share and work things out. Discuss more facts and theories rather than what question are you on?	Better than normal work, think more about something. Enjoyed but wanted to go deeper into topics
Motivation	Wanted to find out more and why it works	More motivated than working out a book as it's no longer individual
Knowledge Gained vs consolidation	Learnt how phi function works and using algebra consolidated it. Was useful to know about primes and time tables for phi function.	Phi made it easier to work out prime numbers and consolidated knowledge of these
Discussion Group vs class	More say in a group discussion. Class wide less useful but could hear what others found out	Easier in a group as could see where went wrong and share. Class discussions were sometimes more of an 'interruption'
Explanations	Could point things out but difficult to explain why and how.	Needed to write things down. Sometimes could not explain patterns
Making discoveries	Happy and pleased when discoveries made. Determined to go further and find out more.	Proud
Decision Making	Made more in phi investigation. Lots of little paths for discovery	n/a
Use of Teacher	Needed help in explaining stuff e.g. powers of 2. Could point stuff out but not sure what it meant	If stuck, teacher could explain in further detail and go further
Investigation Preference	Two said phone: understood more, liked graphs, had an end. One said phi: more freedom and choice	Mobile phones as teenagers like phones and applies to daily life. Did not see relevance of phi
More lessons	All said yes	One said yes. One thought consolidation needed

Table 13: Summary of responses to interview from pupils perceived to cope well.

Groups 1 and 2 above saw the benefits of group discussions, were motivated and enjoyed making discoveries. The investigations consolidated knowledge rather than created new knowledge. Both groups felt they used me as a resource for explaining as they found this difficult and wanted more investigations in the future.

	Responses from Different Groups	
Question Area	Group 3 – Struggled to adapt	Group 4 – coped but got frustrated when stuck
Enjoyment	Enjoyed more as more banter and more interaction	Good, not something specific to do. Could set own limits and figure things out. No exact guidelines
Motivation	Did not feel motivated in phi as unsure what meant to be doing. Did not like algebra in mobile phones	n/a
Knowledge	Gained more. Learnt phi and improved graph work	Improved knowledge of algebra through expressions and examples
Discussion	Exciting to go up the board	Class discussions difficult as lots of shouting out. Groups were better
Explanations	Hard to explain how the graph works when they intersect. Easier to spot rather than explain	n/a
Discoveries	n/a	Excited and proud yet frustrated when dead ends reached
Decision Making	n/a	More in phi. Had to decide what to do next & what theories to try
Use of Teacher	Teacher used a lot. Explain what meant to be doing and how to do sums	Used to explain
Investigation Preference	Phones as did not understand phi	Phone – easier, more to do with something they like but more learnt in phi
More lessons	More lessons as better than normal as something to work out	Yes, but not all. Boring if done all in a row

Table 14: Summary of responses to interview from pupils perceived to cope less well.

Groups 3 and 4 who were perceived as not adapting as well still gave positive responses in their interview saying they had enjoyed the lessons and thought more should be done. They did indicate a greater use of me in helping them to explain and get unstuck. Again, it was indicated that group discussions aided but it was hard to get motivated when the investigation was proving difficult.

The data collection has identified several key issues. Overall, pupils had positive perceptions of the investigations but preferred the less open mobile phones one. Could this be because it was easier, more relevant or delivered more didactically? Syllabus topics were covered and in the mobile phones investigation, new topics were introduced albeit with much steering from me. Syllabus content was probably more consolidated in the phi function investigation. My role was reactionary in the phi function investigation, responding to pupil need, guiding and questioning. However, in the mobile phone investigation I had a more didactic role. This investigation also highlighted the problem of context in problems. Nevertheless, throughout the investigations pupils were using skills of: communication, reasoning, decision making, representation and interconnectedness.

Discussion of my Research

Within this section findings will be discussed, notably their implications and they shall be related back to some of the literature reviewed earlier.

Teacher Role (RQ2)

As stated during the phi function investigation, I found myself developing a reactionary role, responding to pupil questions, neither confirming nor denying their findings. This agrees with what Tanner (1989) stated, "guide but don't eliminate false paths" (p.265). All pupils interviewed indicated they had used me to aid explanations and offer guidance when stuck. Stemn (2008) said the teacher clarifies misunderstandings and asks questions to push further. This, I found myself doing frequently. Some pupils calculated the phi function inaccurately, needing correcting and via questioning one group was pushed to understanding the connection to prime factorisation. Pupils were given knowledge on a need to know basis such as helping pupils who had discovered a pattern with the powers of two to identify them as powers of two. This agrees with what Van Schalwijk et al. (2000) identified, active intervention to enable access to higher levels. However, it could be the case that the literature influenced the way I acted and that I deliberately tried to 'hold back' from teaching. Pritchard (1993) said that choosing the right moment to intervene needed fine judgement, something I do not think I fully mastered. The pupils who did not progress much perhaps needed more intervention and maybe I was veering too much towards constructivism and not giving pupils the necessary help. Van Schalwijk et al. (2000) highlights the pitfalls of constructivism and that a more social constructivist approach is needed. However, throughout both investigations mini-plenaries were used to allow ideas to be shared and discussed which Ofsted (2008) witnessed and identified as a feature of high subject expertise.

One of Cooper's (1990) PGCE students identified difficulties in aiding everyone with questions, something I also found. Several other of his PGCE students identified discipline problems and noisy classrooms. My classroom was noisy but there were no more discipline issues than normal. However, this was a top set class who may be more inclined to adapt to the investigative working style. As previously stated, a more didactic role was developed in the mobile phones investigation. This draws parallels with Van Schalwijk et al. (2000) who changed the amount of teacher input for the second round. Those researchers shifted the emphasis to enable students to make the jump from elementary to advance mathematics. I am not sure I was doing quite the same; with my intervention pupils were doing more advanced mathematics, notably solving linear equations and working with straight line graphs. However, I chose to direct the learning in this way. Tanner (1989) noticed that solution strategies were more transferable if discovered rather than directly taught and whilst there was some discovery, there was more direct teaching which could explain why pupils performed worse in test questions on intercepts and gradients. Cooper (1990) found that some PGCE students struggled in moving from an algorithmic style to the investigative style. However, I did not find this a major issue in the phi function investigation. It was not difficult to be less directive.

It could be said that the investigation itself caused the more didactic teaching style. Cooper (1990) says investigations are divergent whereas problem solving is convergent with solutions. Thus, the mobile phone investigation could be more of a more problem solving task. Thus, this could distort my findings on teaching style. Wells (1985) spoke of the concept of openness, defining it as a "relationship between the problem, the solver, the occasion and even the place, not the problem itself' (p.7) Many pupils saw little value in continuing after tabulating results as indicated by their questionnaires. Maybe they had a different relationship with the problem compared to others. Hunt (2005) says problems and investigations are starting points for schemes of work with the only constraint being how far students take ideas. He states the starting point encourages direct teaching and then the class is brought together to enable a point to be developed by the teacher before exercises for consolidation. This is more similar to the mobile phones task; I brought out ideas of linear equations and straight-line graphs. However, there was a lack of consolidation which could be why pupils performed poorly in the test.

Pupil Perception (RQ3)

Pupils in interviews indicated that they enjoyed the investigations with most saying they preferred the mobile phones task as it was easier, more relevant and more fun. This draws similarities with Cooper (1990), one of his students stated that children preferred traditional work as they knew the aim. In the mobile phones task, pupils had set guidelines; find the cheapest contract. Furthermore, another of Cooper's students stated that pupils panic if they do not understand what is happening and are frustrated by the lack of right answers. This occurred in the phi function investigation. Some did not understand and did not enjoy it and others were frustrated when they spotted patterns but were later proved false. However, this frustration did not deter pupils. They kept persevering and the missing 'eureka moment', as Tanner (1989) describes did not seem to cause them to lose motivation to persevere. One pair in particular continued to explore a multitude of different paths. The lack of answer did appear in the phi investigation. Even pupils spotting challenging patterns such as powers of two felt they had not achieved anything. Perhaps, they were not used to the investigative style. Nevertheless, some pupils from this group indicated they preferred the phi function due to its more open and challenging style. Ofsted (2008) found differences in pupil opinions; some enjoying challenge and some preferring routine exercises, almost exactly what was discovered with these two investigations. A higher preference for the mobile phones investigation could be because as a coursework task it was aimed at foundation and intermediate candidates whereas the phi function was aimed at intermediate and higher candidates. Thus, the mobile phone task was probably more appropriate for year 8 top set students. However, many accessed the phi function with success. I did not study the phi function until undergraduate level and so the capabilities of these young mathematicians greatly impressed me.

The Context of Problems

Several pupils indicated they enjoyed the context of the mobile phones investigation agreeing with Ofsted (2008) who found pupils wanted more contextual problems. This links to Boaler's (1994) work who found that "contexts can motivate, engage interests and encourage confidence" (p.557). Some pupils indicated in interview they saw no relevance of the phi function. It is used in cryptography but this may be beyond their capabilities at a young age. The context caused many problems as previously stated and Boaler (1994) states that "learning transfer occurs only if the mathematics makes sense in the classroom and the real world" (p.557) and that candidates are penalised for using common sense. This could explain why some pupils still linked graphs to mobile phones in the test and why some only considered 'real-life' contracts. This is probably my fault as I updated the original investigation to be more modern and relevant.

Links to Syllabus Topics (RQ4)

Ball (1990) spoke of investigations as bolt-on and irrelevant to topics being studied. These two investigations could be considered ‘bolt-on’ pupils had been studying algebra to which the phi function bears little links. However, the mobile phones task did include some algebra. As previously stated the phi function included some syllabus topics, notably links to factors, primes, multiples and powers. However, pupils had prior knowledge of most of these and so the task was more consolidating knowledge rather than introducing it. I am unsure whether the investigation could be used to introduce this area of mathematics. It would be interesting to study this. Furthermore, the struggle in designing the follow-up test shows this investigation did not lend itself well to syllabus topics. Pupils’ indications that they learnt more about equations and graphs in the mobile phones task, shows the possibility of teaching syllabus topics through investigation. However, this was teacher led. Skovmose (2002) suggested that abandoning the exercise paradigm completely was not the sole answer and an opinion I believe my research supports. Consolidation is needed in some areas. Furthermore, Denscombe (2007) speaks of the difficulty in generalising from a particular case study. My two investigations, whilst providing strong evidence, do not permit me to make strong conclusions as to what topics can be taught. More research would be needed to assess syllabus topics that could be taught through investigation.

What was achieved? (RQ1)

Pupils indicated they enjoyed the group discussion and making discoveries. I witnessed: good communication, representation and systematic working. Stemn (2008) identified these as good features of investigations and also spoke of building on prior knowledge. The phi function built on knowledge of primes and pupils were asked to explain their thinking and invent their own strategies like in Stemn’s project (2008). This is an example of constructivism whereby there is much mental activity on the learner’s part (Van Schalwijk et. al, 2000). There was interaction and discussion with pupils required to make sense of what was happening, an example of social constructivism (Morgan et. al, 2004). Stemn (2008) said investigations help to increase relational understanding. Pupils considered what to do and why and even in the mobile phones task they could see why the graphs and equations were important. However, as Skemp (1976) discussed, relational understanding is hard to assess and it is difficult to know whether my investigations promoted true relational understanding. However, there was evidence of constructivist and social constructivist teaching styles. Thomas (1992) spoke of dangers; pupils generalise too quickly. This was evident in the phi function work where some concluded the result was even based on a few examples. However, in my teacher role I could address this misunderstanding as Stemn (2008) suggested.

Further Questions

The sequence of lessons on investigations has answered several questions but provided many more. The teacher role was more reactionary but it would be interesting to further compare this with respect to a range of varying open problems and maybe with lower attaining sets. Furthermore, the research was conducted entirely with male participants. It would certainly be interesting to compare the same investigation tasks with female participants of similar ability as there would likely be different results. It is difficult to generalise from a narrow sample of only high attaining male participants and so further work could be done comparing genders and ability groups. Certain syllabus topics were taught or reinforced but further work could be done, deciding which topics investigations could teach best. The issue of context was prevalent and so work could be done looking at the impact of context on students' learning. Overall, it seems the lessons were of value to both pupils and I.

Conclusion

The research project looking at the value of investigations has been interesting and worthwhile. I think it is clear that investigations are of value and importance in the mathematics curriculum and it is right that Cockcroft (1982) identified them as one of his six important elements. My research has evidence to suggest that the objective of investigations is to increase relational understanding in pupils by using constructivist and social constructivist approaches. It also suggests that it is implemented by a teacher having more of a reactionary role, that pupils enjoy investigations and that there can be clear links to syllabus topics, in line with the literature reviewed.

On a professional level I think this project has been very important so early in my career. I have been able to establish ways of getting pupils to think at higher levels through good questioning techniques and have noticed the importance of deciding when and when not to teach. The project has also highlighted issues of consolidating what has been learnt and the problems of context.

With regards to students' learning my research suggests that students enjoy this style of working and that it can increase their understanding. However, it also shows there needs to be a balance between investigation and consolidation work to ensure that pupils both gain and retain knowledge and understanding.

Also, some of my work shows that not all of the pupils in a class fully participate and so more work could be done in aiding teachers to develop techniques to address this and encourage participation and investigation during investigation lessons.

I think it is clear to note that investigations are of great value to pupils and their learning but more work needs to be done to ensure they are used correctly and act as a vehicle to increase mathematical understanding.

References

Ball, G. (1996), The Use of Investigative Activities in the Teaching of Mathematics at Key Stage 3 in Welsh Secondary Schools. *Education Today,* 46(2), 24-29

Bassey, M. (1999), *Case Study Research In Educational Settings.* Milton Keynes: Open University Press

Bell, J. (2010), *Doing Your Research Project,* (5th ed.), Maidenhead, Open University Press

Boaler, J. (1994), When do girls prefer football to fashion? An analysis of female under achievement in relation to realistic mathematics contexts. *British Educational Research Journal*, 20 (5), 551- 564.

Cockroft, WH, (1982) – The Cockcroft Report: Mathematics Counts: Report of the committee of Inquiry into the Teaching of Mathematics in Schools under the Chairmanship - HMSO

Cohen, L., Manion, L., & Morrison, K. (2007), *Research Methods In Education* (6th Ed.) – London: Routledge

Cooper, B. (1990), PGCE students and investigational approaches in secondary maths, *Research Papers in Education*, 5(2), 127-151

Denscombe, M. (2007), *Good Research Guide: For Small Scale Social Research Projects*, Maidenhead, Open University Press

Driver, D. (1988), Investigative Mathematics in Schools, *Mathematics in School,* 17(1), 2-4, The Mathematical Association

Goulding, M. (2011) – Chapter 3 – Pupils Learning Mathematics in S. Johnston-Wilder, P. Johnston-Wilder, D. Pimm & C. Lee – *Learning to Teach Mathematics in the Secondary School* – 3rd ed. (pp. 36-53) - Routledge

Hopkins, C. (1995) Open Teaching, *Mathematics Teaching*, 150, 41-43

Hunt, D. (2005), In Praise of GCSE Coursework in Mathematics, *Mathematics in School*, 34(5), 12-13, The Mathematical Association

Jaworski, B. (1996), Investigating Mathematics Teaching: A Constructivist Enquiry, London: Routledge

Morgan, C., Watson, A. & Tikly, C. (2004) – *Mathematics: Teaching School Subjects 11-19* – London: Routledge Falmer

Nisbet, J. & Watt, J. (1984), Case Study. In J. Bell, T. Bush, A. Fox, J. Goodey and S. Goulding (eds), *Conducting Small Scale Investigations in Educational Management.* London: Harper & Row, 79-92

Office for Standards in Education (OFSTED). (2008). *Mathematics: Understanding the Score – Messages from Inspection Evidence*. London: Her Majestey's Staionary Office (HMSO)

Oppenheim, A.N. (1992), Questionnaire Design, Interviewing and Attitude Measurement, London: Pinter

Pritchard, C. (1993), Challenges of Revised Higher Investigations, *Mathematics in School*, 22(1), 37-39, The Mathematical Association

Robson, C. (2002), *Real World Research,* (2nd ed.), Oxford: Blackwell

Skemp, R. (1976), Relational Understanding and Instrumental Understanding, *Mathematics Teaching* 77, 20-26, ATM

Skovmose, O. (2002), Landscapes of Investigation. In L. Haggarty, *Teaching Mathematics in Secondary Schools* (115-128), London: RoutledgeFalmer

Stemn, B.S. (2008), Building middle school students' understanding of proportional reasoning through mathematical investigation, *Education 3-13: International Journal of Primary, Elementary and Early Years Education*, 36(4), 383-392

Tanner, H. (1989), Managing Perceptions through Action Research: introducing investigations and problem-solving — a case study, *School Organisation*, 9(2), 261-269

Thomas, G. (1992), Some Dangers in the Use of Investigations in Mathematics Teaching, *Teaching Mathematics & Its Applications,* 11(2), 76-78

Van Schalkwijk, L., Bergen, T., & Van Rooij, A. (2000), Learning to Prove by Investigations: A Promising Approach in Dutch Secondary Education, *Educational Studies in Mathematics*, 43(3), 293-311

Watts, M. & Ebbutt, D, (1987), More than the Sum of the Parts: Research Methods in Group Interviewing, *British Educational Research Journal,* 13(1), 25-34

Wells, D. (1985), Cockcroft, Problems, Investigations and Confusion, *Mathematics in School*, 14(1), 6-9, The Mathematical Association

Wells, D. (1995), Investigations and The Learning of Mathematics, Mathematics Teaching, 150, 35-40

Appendix 1 Initial Questionnaire

Pupil Questionnaire into Perceptions of Mathematical Investigations
Answer the following questions honestly and truthfully. Circle 1 if you strongly agree with the statement, 2 if you agree with the statement, 3 if you disagree and 4 if you strongly disagree

1. I learn well by working through textbook exercises on a topic
   1 2 3 4

2. I enjoy doing mathematical investigations
   1 2 3 4

3. I feel confident tackling open problems, not knowing where the work is leading
   1 2 3 4

4. I like to know the specific mathematical topic we are studying
   1 2 3 4

5. Doing investigative work helps to increase my understanding of a topic
   1 2 3 4

6. Discussing ideas and communicating increases my understanding
   1 2 3 4

7. I enjoy discovering new things in mathematics by myself
   1 2 3 4

8. I like spotting patterns and trying to explain them
   1 2 3 4

9. I like to understand the mathematics I am doing
   1 2 3 4

10. Getting the right answer is more important than understanding how to get it
    1 2 3 4

11. My teacher's role is to give me rules and methods to follow
    1 2 3 4

12. My teacher's role is to help me construct my own knowledge
    1 2 3 4

13. Investigations should be part of the mathematics curriculum
    1 2 3 4

14. Explain briefly why you think we do mathematical investigations in the classroom

____

____

Appendix 2 Outline of Phi Function Investigation Given to Students

The Phi Function – An Investigation

For any positive integer (whole number) n, the Phi function Φ(n) is the number of positive integers below the integer n, which are co-prime to n (no common factors with n).

e.g. Φ(16) = 8

1, 3, 5, 7, 9, 11, 13, 15 are all less than 16 and have no common factors with 16 (except 1). i.e. there are 8 numbers less than 16 which are co-prime to 16.

Part 1

Find the value of:

i) Φ(3)
ii) Φ(8)
iii) Φ(11)
iv) Φ(24)

What method are you using to calculate Φ(n)? Do you need to consider every number? Try to explain if you have found an easier method

From the 6 results you now have can you make any generalisations or predictions about Φ(n). If you can, test your predictions for some other values of n and then try to explain or prove these.

If not, calculate the phi function for some other values of n. Order your results systematically and look to see if there is a relationship between n and the value of Φ(n) for certain values of n. You may wish to consider certain types of number!

Again, try to explain/prove results.

What we are looking for is some sort of generalisation about the value of Φ(n) for certain integers. Look for patterns!!

Part 2

Does

i) Φ(7 x 4) = Φ(7) x Φ(4)
ii) Φ(6 x 4) = Φ(6) x Φ(4)

Investigate whether the phi function of a product is always equal to the product of the phi functions of its components. i.e. does Φ(m x n) = Φ(m) x Φ(n)

Again, do some calculations. Organise them systematically and try to spot patterns. Test your ideas and try to explain them

Appendix 3 Outline of Mobile Phones Function Investigation Given to Students

Mobile Phones

Mr Corner is trying to buy a new iPhone. Can you help him to decide which contract to get. These are 3 different schemes I am considering:

T-Mobile: The handset costs £300 with a monthly tariff of £10.50

Vodafone: The handset is free but the monthly tariff is £51

Orange: The handset costs £149.99 and the monthly tariff is £20.50

Assume all three options are comparable in terms of minutes, texts and data allowance per month.
Can you help me to find one which one to choose?

What does it depend on? What methods are you using?

ASSUME ALL I WANT TO DO IS MINIMISE COST

Appendix 4 End of Investigational Work Questionnaire and Test

As we reach the end of our sequence of lessons doing investigation work, I wish to gather some thoughts and opinions from you on the topic. This questionnaire asks you your thoughts and opinions on the two different investigations. It will also test you on some knowledge you may have gained or consolidated throughout the lessons.

These questions will briefly ask some questions about the two different investigations

1. Please indicate which of the two investigations you preferred
   a. The Phi Function Investigation ☐
   b. The Mobile Phones Investigation ☐
2. Briefly explain as to why you have chosen the you did investigation above

   ____

   ____

In the following sections for questions with a rating scale, circle 1 if you strongly agree, 2 if you agree, 3 if you disagree, and 4 if you strongly disagree

The Phi Function:

3. I coped well with this investigation

   1 2 3 4
4. The openness of the problem appealed to me

   1 2 3 4
5. It was easy to make progress with this investigation

   1 2 3 4
6. What does it mean for two numbers to be co-prime?

   ____
7. What is $\Phi(n)$ if n is prime? Why?

   ____
8. Without doing any calculations would $\Phi(4x3) = \Phi(4) \times \Phi(3)$? Why?

   ____
9. Without doing any calculations would $\Phi(4x2) = \Phi(4) \times \Phi(2)$? Why?

   ____

10. When does $\Phi(n \times m) = \Phi(n) \times \Phi(m)$?

    ____

11. How is prime factorisation of a number important to calculating the phi function for a number?

    ____

Mobile Phones:

12. I coped well with this investigation

    1 2 3 4

13. It was easy to make progress with this investigation

    1 2 3 4

14. I liked the context of the problem

    1 2 3 4

15. I was satisfied that the investigation had a solution

    1 2 3 4

16. I saw little point in continuing when we had already solved the initial problem

    1 2 3 4

17. Which of these equations will produce the steepest line graph? Why?

    y= 3x+4

    y= 2x +15

    y= 9x + 1

    ____

18. What is the relevance of the numbers 4, 15 and 1 in the above?

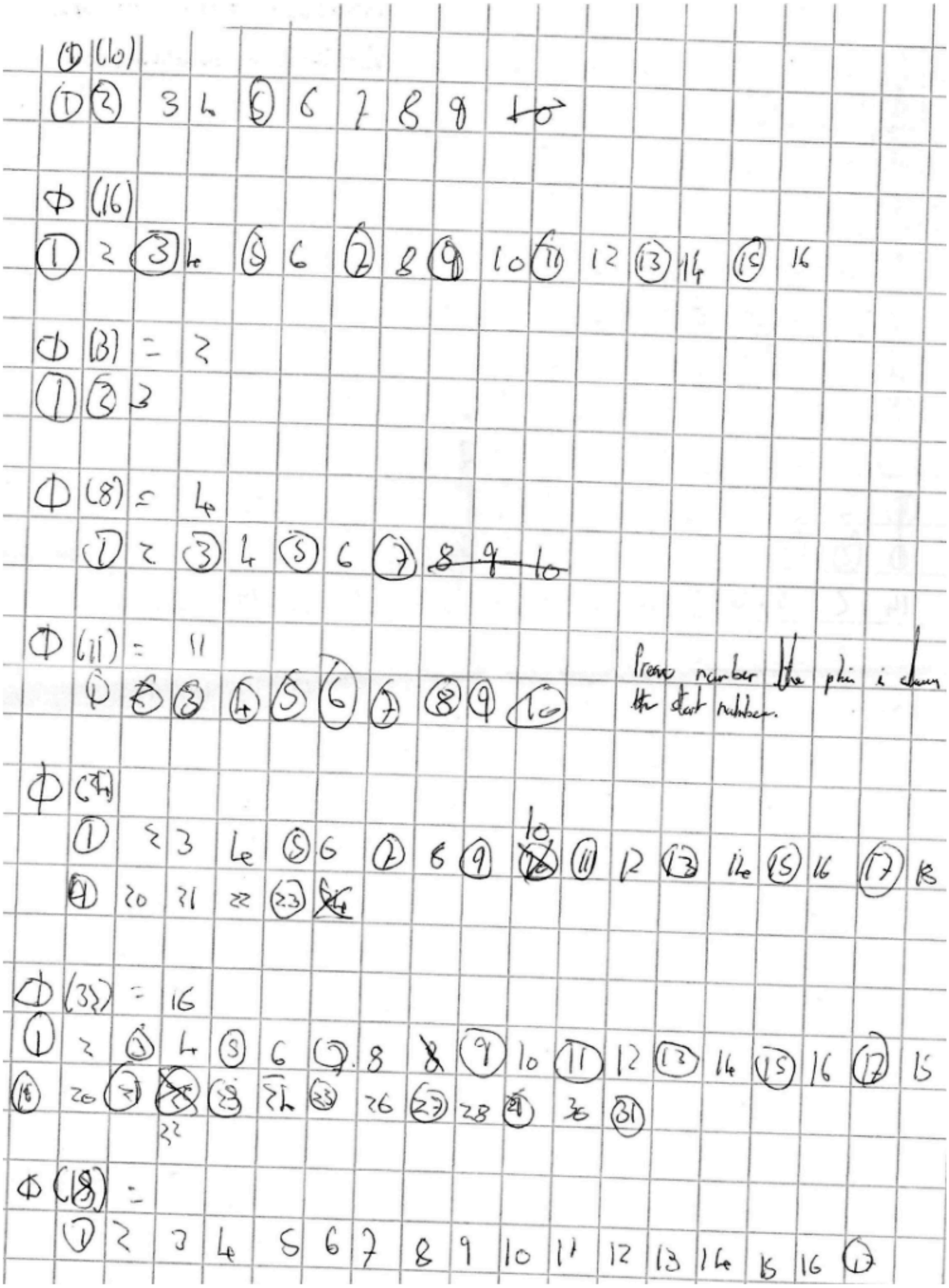
    ____

19. What is significant about the points where lines on a graph intersect?

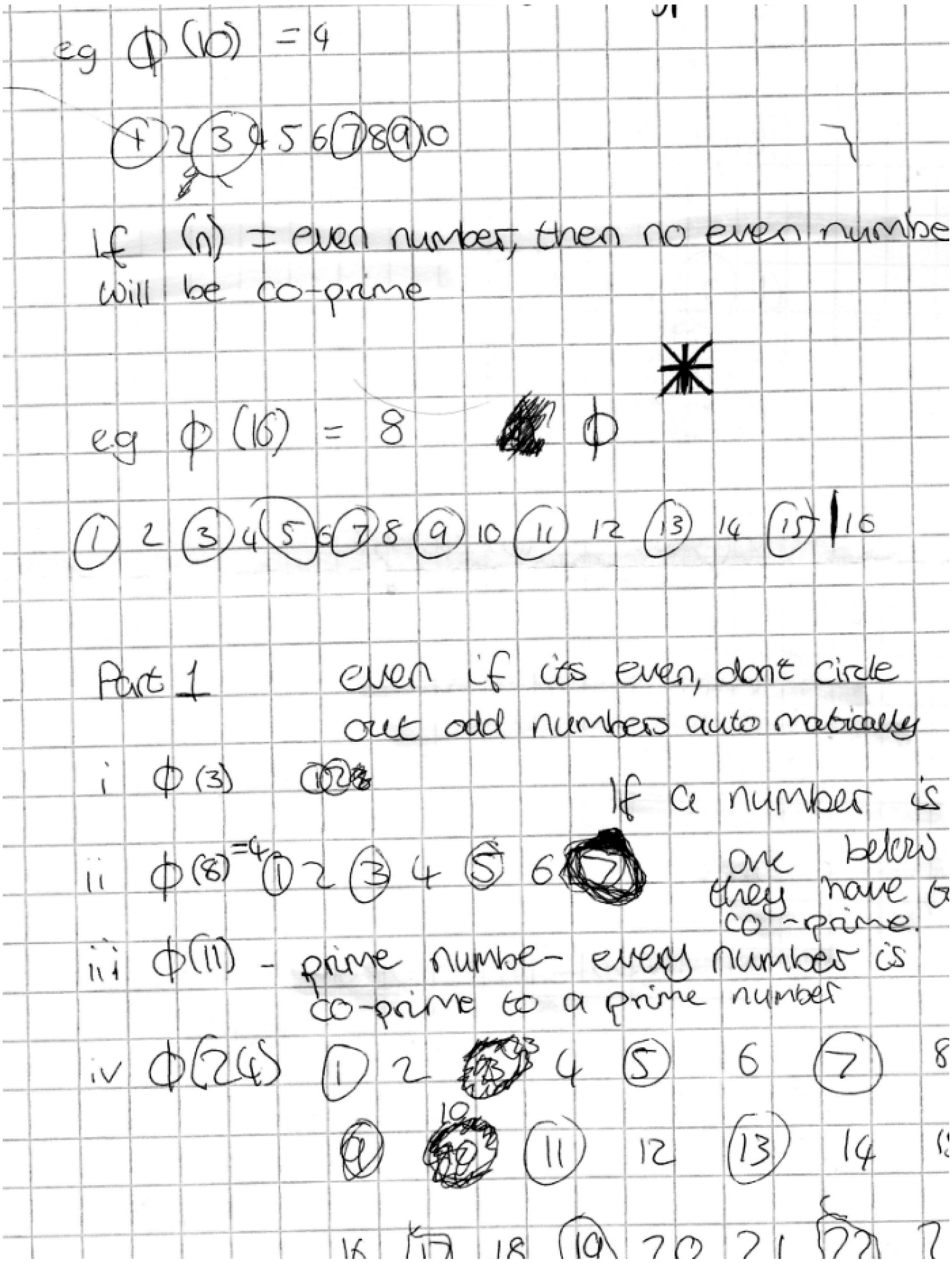
    ____

20. Without using a graph, how else could you find out where they would intersect?

Appendix 5 Example of a Pupil who has worked systematically on the Phi Function Investigation


Φ (16)
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
Φ (3) = 2
Φ (8) = 4
Φ (11) = 11
Prime number the phi is ... the start number.
Φ (32) = 16
Φ (18) =

Appendix 6 Example of a Pupil who has considered Odd and Even Numbers and Other Types of Number


eg ϕ(10) = 4

1 2 3 4 5 6 7 8 9 10

If (n) = even number, then no even number will be co-prime

eg ϕ(16) = 8   ϕ

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

Part 1   even if its even, dont circle out odd numbers automatically

i ϕ(3)

If a number is one below they have co-prime.

ii ϕ(8) = 4 1 2 3 4 5 6 7

iii ϕ(11) - prime number - every number is co-prime to a prime number

iv ϕ(24) 1 2 3 4 5 6 7 8

9 10 11 12 13 14

16 17 18 19 20 21 22

Appendix 7 Smart Board File of Questions asked of Pupils following the First Lesson

The Phi Function - An Investigation

Remember the phi function written Φ(n) is a counting function which looks at all numbers less than a integer and counts all those co-prime to the integer i.e. has a highest common factor of 1 with the integer.

Continue to investigate the phi function today.

Things to consider that we briefly discussed yesterday: prime numbers, odd numbers, even numbers, powers of primes..... or any other pattern you spot. Maybe work through numbers systematically to try and spot patterns.

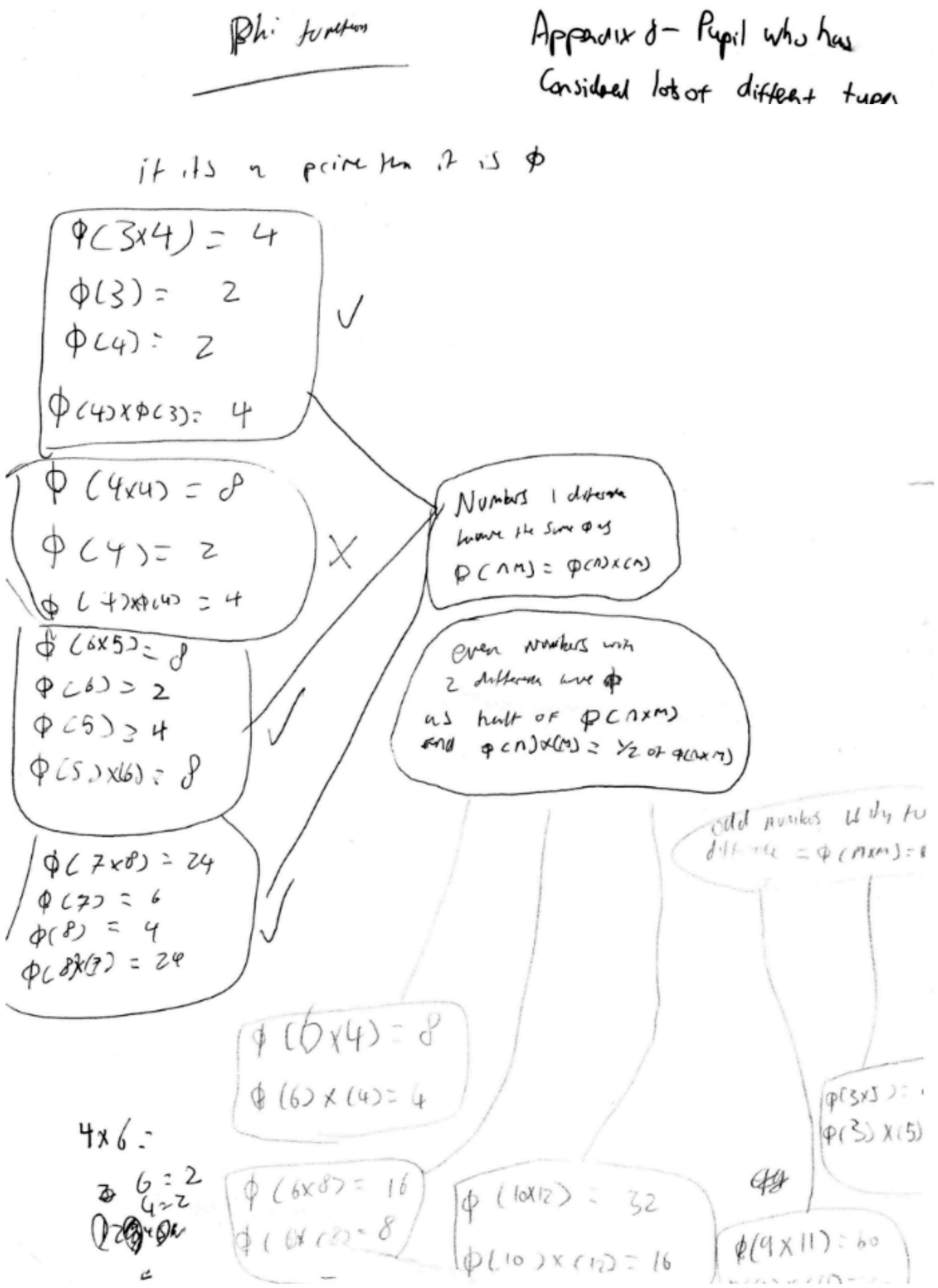
As ever if you spot a pattern, test it out for other numbers in an attempt to prove your claim. If it works, try to explain but don't worry if you can't!

Once you have investigated the phi function thoroughly look at either of these:

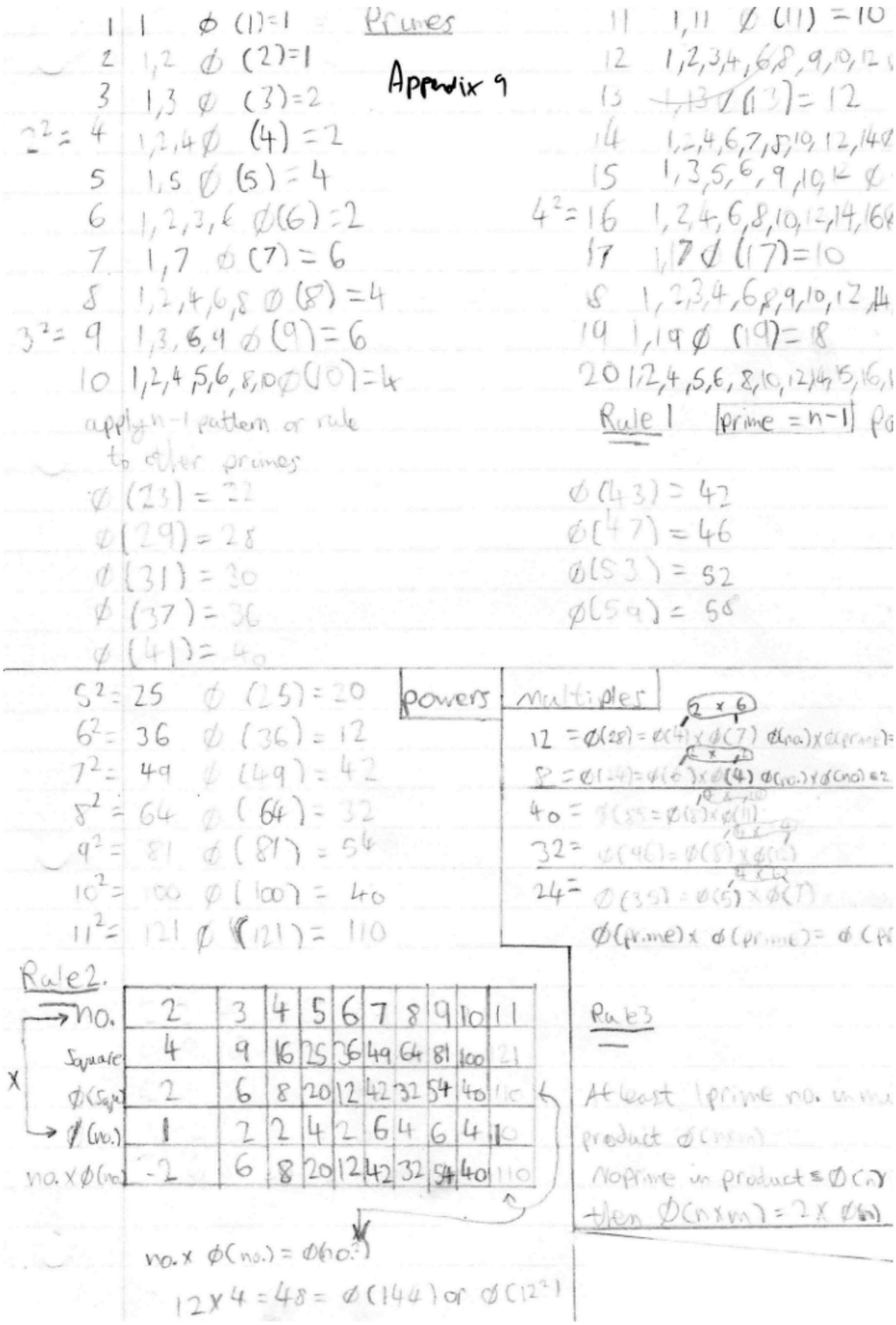
1. Does Φ(nxm) = Φ(n)xΦ(m). Look into this. Again work through systematically and look for patterns, before trying to explain

2. How would we calculate Φ(n) for larger numbers e.g. Φ(100) or Φ(99) or even bigger? Do we need to work through every single number lower than it? Try to explain what numbers you need to consider for certain integers

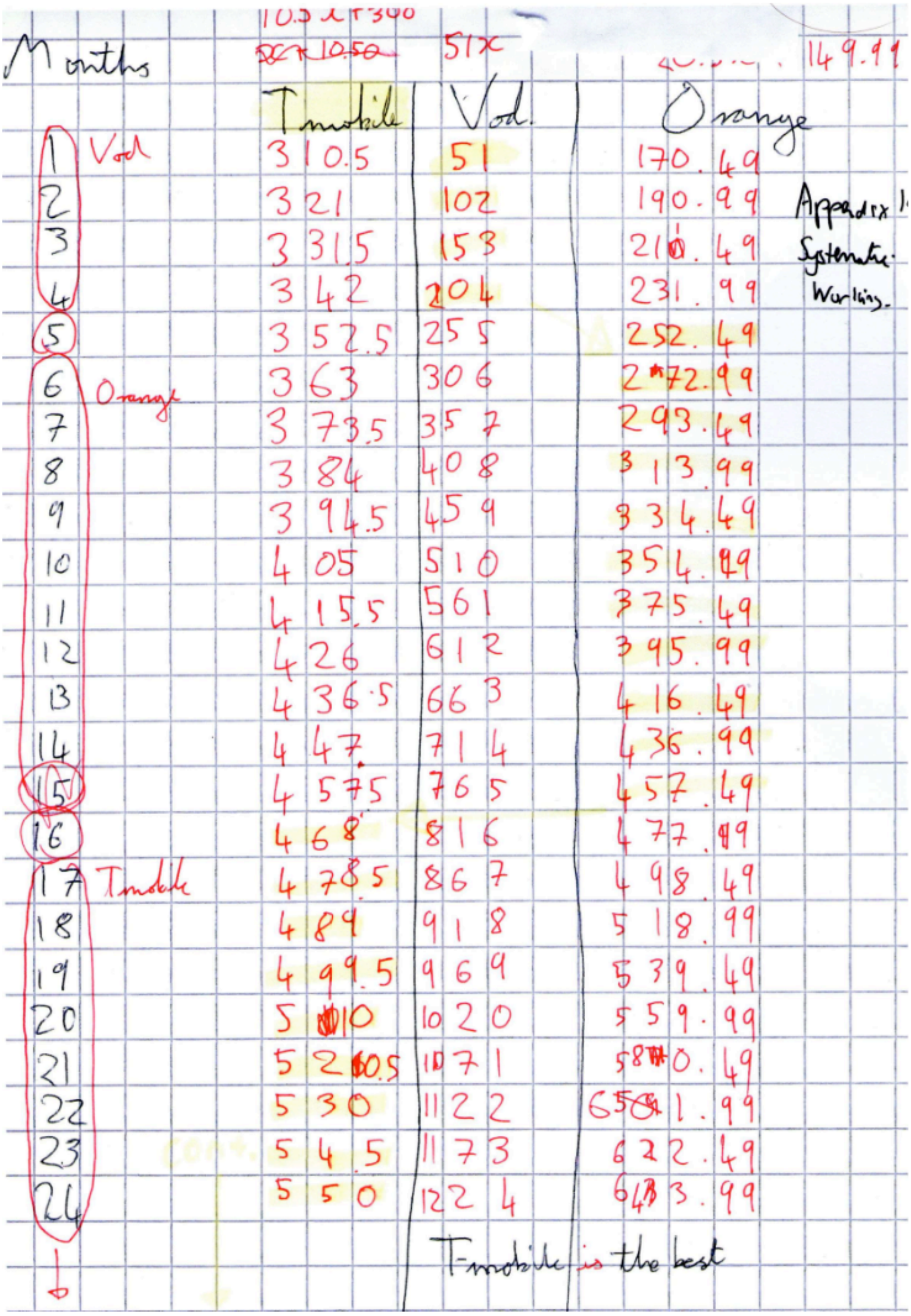
Appendix 8 – Example of a Pupil who has considered lots of different types of number



Appendix 9 Example of a Pupil working on Powers & Multiples



Appendix 10 Example of a Pupil working systematically on the Mobile Phones Investigation



10.5x + 300

~~30x + 1050~~

51x

149.99

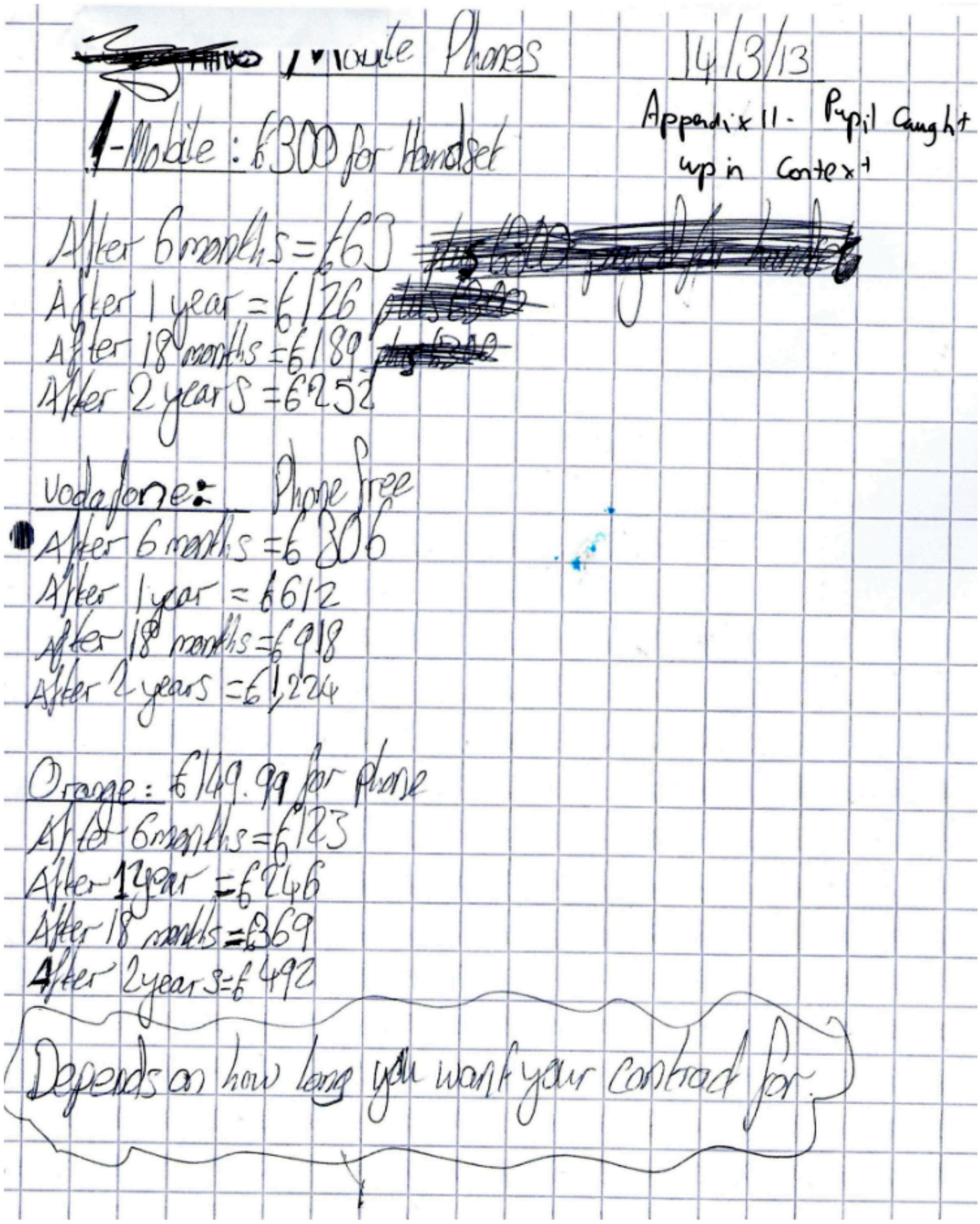
Months		T mobile	Vod.	Orange
1	Vod	310.5	51	170.49
2		321	102	190.99
3		331.5	153	210.49
4		342	204	231.99
5		352.5	255	252.49
6	Orange	363	306	272.99
7		373.5	357	293.49
8		384	408	313.99
9		394.5	459	334.49
10		405	510	354.99
11		415.5	561	375.49
12		426	612	395.99
13		436.5	663	416.49
14		447	714	436.99
15		457.5	765	457.49
16		468	816	477.99
17	Tmobile	478.5	867	498.49
18		489	918	518.99
19		499.5	969	539.49
20		510	1020	559.99
21		520.5	1071	580.49
22		530	1122	65[illegible]1.99
23		545	1173	622.49
24		550	1224	643.99

Appendix 1

Systematic Working.

T-mobile is the best

Appendix 11 Example of a Pupil caught up in the Context of the Mobile Phones Investigation


[redacted] Mobile Phones 14/3/13

Appendix 11 - Pupil Caught up in Context

1-Mobile: £300 for Handset

After 6 months = £63 [redacted]
After 1 year = £126 [redacted]
After 18 months = £189 [redacted]
After 2 years = £252

Vodafone: Phone free
After 6 months = £306
After 1 year = £612
After 18 months = £918
After 2 years = £1,224

Orange: £149.99 for phone
After 6 months = £123
After 1 year = £246
After 18 months = £369
After 2 years = £492

Depends on how long you want your contract for.

Appendix 12 Questions for Pupils to Consider on the Mobile Phones Graph

le phones.notebook Appendix 12– Questions for Pupils to Conside April 27, 2013

Things to Consider and Investigate

Are there any relationships between the equations of the lines and how they are represented graphically?

Which graphs are steeper? why?

How can we use the graphs to solve our original problem?

How many points do you need to plot when drawing a straight line graph?

1

Appendix 13 An Example of the Graph drawn